Introduction

- This paper draws on work in philosophy of logic and foundations of mathematics to consider debates over the use of mathematics in economics, especially those concerning claims that economics is too "formalistic."
- Vela Velupillai and co-authors Thomas Boylan and Paschal O'Gorman: classical mathematics is inappropriate for use in economics and constructive methods, such as numerical ones, should be adopted instead; current typical economics methodology is too "formalistic."
- I consider:
  - 1) how work in philosophy of mathematics and logic bears on the claims of Velupillia and Boylan/O'Gorman
  - 2) what Roy Weintraub's (2002) distinction between formalism in foundations and formalism as axiomatization tells us about these arguments
  - 3) what these investigations into philosophy of applied mathematics can tell us about the charge of "formalism" more generally.
  - I argue that overall, while the arguments are aptly understood as showing a problem with some contemporary economics, those problems have to do not with mathematical foundations -- i. e., with formalism about the nature of mathematical entities but rather about choosing appropriate mathematical tools for given purposes and, more centrally, with broader questions about what it means to "axiomatize" a science.

Background

- Classical mathematics: uses first-order logic, including LEM; allows non-constructive proofs, including for existence claims; functions typically defined in terms of sets.
- Intuitionism: variety of forms. Idea that objets are creations of the human mind, so one cannot claim "P or Q" unless one has a proof of P or a proof of Q.
- Intuitionists generally regard the use of LEM as a logical law in mathematics -- and thus classical mathematics itself -- as illegitimate in some way.
- Adopting intuitionistic logic leads to a different conceptualization of many fundamental concepts in mathematics, especially in fields like analysis. In prominent strands:
  - real numbers are constructed out of "free choice" sequences.
  - functions must be constructed, rule-based.
  - every function is a continuous function.
  - fixed-point theorems are not valid unless the proof gives a method for finding the point.
- Formalism in foundations of mathematics: mathematical objects are understood in terms of formal symbols subject to certain rules of manipulation.
- Hilbertian formalism: natural numbers refer to real entities; abstractions in mathematics are to be understood according to whether they are truth-preserving in their implications for these real entities.
- Other forms: all mathematical objects are understood in terms of symbols with definitions.
- Gödel's theorems show there is no way to lay out axioms so mathematics can be shown to be consistent; for any axiomatization there will always be undecidable statements.
- Formalism usually uses first-order logic and is associated with classical mathematics.
- Distinctions are salient in contexts like GET: General Equilibrium Theory.

- "if agents are rational, self-interested, and well informed, and if they interact only through voluntary exchange in a perfectly competitive market, then a general equilibrium exists, which is Pareto efficient."
- "every Pareto efficient outcome is a general equilibrium of voluntary exchanges among rational and self-interested agents, given the proper initial distribution of resources among the agents" (Hausman).
- Uses real analysis, non-constructive fixed point theorems.

The arguments: formalism, fit, and physics

- Velupillai:
  - (1) the standard approach in GET frames prices as taking on "non-negative" values -- that is, values along the positive real number line. But this is odd, he says, given that actual prices are necessarily integers and fractions. Real analysis is a bad **fit**.
  - 2) economics lacks the "compression quality" of **physics** in which a small number of principles allow prediction of a wide range of facts.
  - following from one and two, the role of the econ theorist is more to track a complex set of facts than to unify and simplify through the kinds of kinds of equations typical of GET.
- Boylan and O'Gorman:
  - (1) Economic phenomena are social phenomena; for that reason, they are "abstract, open-ended, [and] constructed by the human mind." Intuitionistic math "tailor-made" for this, a great **fit**.
  - 2) economics "lacks the sophisticated experimental resources which are distinguishing characteristic of advanced **physics**."
  - 3) a syntactic or **formalistic** approach to axiomatization in which concepts are defined abstractly and not in terms of referents is inappropriate: existence proofs show only the possibility of existence.
  - Example of 3) in GET: definition of preference, rationality etc; the proof in GET which ends in the equilibrium result merely shows there is one, not how to find it.

Analysis and evaluation

- Four thoughts:
- 1) while formalism is one way of supporting classical mathematics and responding to the metaphysical challenges of mathematical ontology, it is not the only way.
  - Neo-realism, if-then-ism, naturalism.
  - Penelope Maddy: LEM can be seen as successful idealization over core logic.
  - So: it's mistaken to cast "formalism" as the only relevant alternative to intuitionism and constructivism.
- 2) "Fit": not typically the issue in how math gets applied but rather what works.
  - If space turns out to be discrete, would we stop using Pythagorean theorem?
  - Statistics is used to model discrete populations and risk for people with very sophisticated math embedded in classical understanding of functions, real numbers, etc.

- 3) Differences among a) formalism in foundations of math, b) formalism as a way of approaching what mathematics should be like, and c) formalism as a way of approaching the axiomatization of a science are salient here.
  - Roy Weintraub: distinguish between Hilbertian Finite Program in Foundations of Arithmetic (FPFA) and Hilbertian "Axiomatic Approach" (AA). The latter is a way of approaching what mathematics should be like: proceed by laying down definitions and postulates.
  - As I see it, Boylan and O'Gorman arguments mostly apply not to FPFA but rather to AA and to the related question of how to axiomatize science.
  - This is because they offer a criticism of the method of defining concepts (like that of preference) in a formal way cut off from folk understandings of concepts.
- 4) The previous point most interesting when seen in the light of the difference from physics.
  - Idea would be that in the absence of powerful experiments, the use of syntactic axiomatization is too risky: we go astray without a proper check on our methods.
  - Velupillai: the economist should be engaged in a modest task of keeping track of things.
  - This framing echoes longstanding debates over excessive formalism in economics, and raises deep epistemological questions.

Conclusions

- The real issues in this debate have little to do with mathematical foundations or philosophy of mathematics and much to do with 1) what kinds of mathematical tools are suited to which purposes and 2) how concepts should be linked to reality in scientific axiomatizations. These are both crucially important and complex questions, but they are also questions that have little to do with the kind of "formalism" that concerns what numbers are.
- What is called for is more sophisticated work in the philosophy of applied mathematics and questions of axiomatization and justification, especially in domains like economics where methodological questions raise different issues from those in contexts like physics.

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