Anti-Formalism in Philosophy of Economics: Mathematical Foundations or Mathematical Style? by Patricia Marino

Introduction

- This paper draws on work in philosophy of logic and foundations of mathematics to consider debates over the use of mathematics in economics, especially those concerning claims that economics is too "formalistic."
- Vela Velupillai and co-authors Thomas Boylan and Paschal O'Gorman: classical mathematics is inappropriate for use in economics and constructive methods, such as numerical ones, should be adopted instead; current typical economics methodology is too "formalistic."
- I consider:
  - 1) whether, and how, these arguments are meant apply to economics in virtue of something specific about economics itself -- e. g. its status as a social science.
  - 2) how work in foundations of logic and mathematics bear on these claims.
  - 3) what is the relation between foundations of math and claims that econ is too formalistic?
- I argue that while the arguments are aptly understood as showing a problem in the methodology of some contemporary economics, the problems have to do not with mathematical foundations -- e. g., with formalism about the nature of mathematical entities -- but rather with mathematical style -- that is, with decisions about which mathematical tools and concepts are suited to which purposes -- and with what it means to "axiomatize" a science.

Foundations of mathematics background

- Classical mathematics: uses first-order logic, includes LEM, allows non-constructive proofs, functions typically defined in terms of sets.
- Intuitionism: variety of forms. Idea that objets are creations of the human mind, so one cannot claim "P or Q" unless one has a proof of P or a proof of Q.
- Intuitionists generally regard the use of LEM as a logical law in mathematics -- and thus classical mathematics itself -- as illegitimate in some way.
- Adopting intuitionistic logic leads to a different conceptualization of many fundamental concepts in mathematics, especially in fields like analysis. In Brouwerian intuitionism:
  - real numbers are constructed out of "free choice" sequences.
  - functions must be constructed, rule-based.
  - every function is a continuous function.
  - fixed-point theorems are not valid unless the proof gives a method for finding the point.
- Formalism in foundations of mathematics: mathematical objects are understood in terms of formal symbols subject to certain rules of manipulation.
- Hilbertian formalism:
  - natural numbers refer to real entities.
  - abstractions in mathematics are to be understood according to whether they are truth-preserving in their implications for these real entities.
- Formalism usually uses first-order logic and is associated with classical mathematics.

The arguments

- Distinctions are salient in contexts like GET: General Equilibrium Theory.
- "if agents are rational, self-interested, and well informed, and if they interact only through voluntary exchange in a perfectly competitive market, then a general equilibrium exists, which is Pareto efficient."
- "every Pareto efficient outcome is a general equilibrium of voluntary exchanges among rational and self-interested agents, given the proper initial distribution of resources among the agents" (Hausman).
- Uses real analysis, non-constructive fixed point theorems.
- Velupillai: using non-constructible fixed-point theorems to prove equilibrium results where equilibria cannot be directly computed is an "unnatural" and "ineffective" use of mathematics.
- the "modern masters of mathematical general equilibrium theory" (859) embraced the "formalist school of mathematics" and thus non-constructive mathematical frameworks like classical real analysis; this is lamentable. (E. g., Debreu.)
- Boylan and O'Gorman: non-computable results render explanations "economically vacuous"
- framing the foundations of mathematics differently will lead to different mathematics being used, which will in turn radically alter the framings of economic concepts we use.

Why intuitionistic logic and constructive mathematics in economics?

- If classical mathematics is thought to be ill-suited to use in economics, is this because it is unjustified as mathematics generally, as some intuitionists would have it, or whether it is thought to be ill-suited to economics in particular? If the latter, why?
- Velupillai: cites Arthur Lesk on the "ineffectiveness" of mathematics in biology. Argues:
  - 1) the phenomena being theorized about in economics take on discrete integer-based values, and numerical methods properly reflect that.
  - 2) economics lacks the "compression quality" of physics in which a small number of principles allow prediction of a wide range of facts.
  - 3) following from one and two, the role of the econ theorist is more to track a complex set of facts than to unify and simplify through the kinds of kinds of equations typical of GET.
- Boylan and O'Gorman: from the "strict intuitionist view" that classical math is illegitimate, it follows that the economics in areas like GET would have to radically change. I agree; I set this aside. With respect to a "pragmatic" intuitionism in which we can choose our approach:
  - 1) Economic phenomena are social phenomena; for that reason, they are "abstract, open-ended, [and] constructed by the human mind." Intuitionistic math "tailor-made" for this.
  - 2) economics "lacks the sophisticated experimental resources which are distinguishing characteristic of advanced physics."
  - 3) a certain formalist and syntactic approach to axiomatization in which concepts are defined abstractly and not in terms of referents is inappropriate in economics: existence proofs show only the possibility of existence.
- Example of 3) in GET: definition of preference, rationality etc. Existence of equilibrium.
- We can summarize this way:
  - 1) Economic reality is finite and rational-number based so its mathematics should be too; thus intuitionistic foundations are to be preferred. Issue is "fit."

- 2) Economics is different from physics, either because it lacks compression (Velupillia) or because we can't carry out sophisticated experiments (Boylan and O'Gorman).
- 3) The particular kind of "formalist" axiomatization in which mathematics is applied to uninterpreted abstract concepts to derive theorems is wrong for economics, partly because of problems to do with the existence: what is claimed to exist mathematically may not exist in reality.

Analysis

- Four points:
- 1) while formalism is one way of supporting classical mathematics and responding to the metaphysical challenges of mathematical ontology, it is not the only way.
  - Neo-realism, if-then-ism, naturalism.
  - Penelope Maddy: LEM can be seen as successful idealization over core logic.
  - So it's mistaken to cast "formalism" as the only relevant alternative to intuitionism and constructivism.
- 2) "Fit": not typically the issue in how math gets applied but rather what works.
  - If space turns out to be discrete, would we stop using Pythagorean theorem?
  - Statistics is used to model discrete populations and risk for people with very sophisticated math embedded in classical understanding of functions, real numbers, etc.
  - Even Lesk, above, not advocating for different foundations of math; focuses on usefulness.
- 3) Differences among a) formalism in foundations of math, b) formalism as a way of approaching what mathematics should be like, and c) formalism as a way of approaching the axiomatization of a science are salient here.
  - Roy Weintraub: distinguish between Hilbertian Finite Program in Foundations of Arithmetic (FPFA) and Hilbertian "Axiomatic Approach" (AA). The latter is a way of approaching what mathematics should be like: proceed by laying down definitions and postulates.
  - As I see it, Boylan and O'Gorman arguments mostly apply not to FPFA but rather to AA and to the related question of how to axiomatize science.
  - This is because they offer a criticism of the method of defining concepts (like that of preference) in a formal way cut off from folk understandings of concepts.
- 4) The previous point most interesting when seen in the light of the difference from physics.
  - Idea would be that in the absence of powerful experiments, the use of syntactic axiomatization is too risky: we go astray without a proper check on our methods.
  - Velupillai: the economist should be engaged in a modest task of keeping track of things.
  - This framing echoes longstanding debates over excessive formalism in economics, and raises deep epistemological questions.
  - But this framing focuses not on foundations of math but rather how to use math, mathematical style.

Conclusion

- I've argued:
  - The arguments are best interpreted as being not about foundations of mathematics, but rather about mathematical style -- which tools and concepts should be used.

  - Claims about excessive formalism are best understood as claims about how economics is axiomatized rather than claims about formalism as a way of understanding mathematical objects.
- The question of whether, and when, axiomatizing a science formalistically --in the sense of the AA articulated by Weintraub -- leads us astray is a deep and important one, related to ongoing claims like that of Pratten (2004) of economics being "too formalistic."
- This study suggests the importance of attending to distinctions among foundation of mathematics, branches of mathematics, ways of applying mathematics, and ways of bringing in empirical testing.
- Although I am challenging certain details of the approaches of Velupillai, Boylan, and O'Gorman, I support the essence of their critique, and I agree with them as well that the problem is one in the philosophy of applied mathematics, an area ripe for further research.

Interesting quote:
"Mixing the connection between mathematics and economics with the idea of formalism in economics is explosive for those who try to reconstruct the history of economics in the twentieth century. It is easier to reject "Formalism" than it is to come to terms with the Axiomatic Approach. I suspect this is the reason why the related topic of the proper role of mathematics in the social sciences is so very controversial. What is at stake is, to put it starkly, the concept of scientific truth -- economic truth -- itself." (Weintraub 2002).

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